$$s(\theta)=\begin{cases} 0.2991\left(\dfrac{\theta}{\pi/4}\right)^5-1.2277\left(\dfrac{\theta}{\pi/4}\right)^4+1.4288\left(\dfrac{\theta}{\pi/4}\right)^3, \quad 0\le\theta\le\pi/4 \\ -0.0580\left(\dfrac{\theta-\pi/4}{\pi/4}\right)^5+0.2679\left(\dfrac{\theta-\pi/4}{\pi/4}\right)^4-0.4911\left(\dfrac{\theta-\pi/4}{\pi/4}\right)^3 \\ \qquad -0.0893\left(\dfrac{\theta-\pi/4}{\pi/4}\right)^2+0.8705\left(\dfrac{\theta-\pi/4}{\pi/4}\right)+0.5, \quad \pi/4\le\theta\le\pi/2 \\ 0.0580\left(\dfrac{\theta-\pi/2}{\pi/4}\right)^5-0.0223\left(\dfrac{\theta-\pi/2}{\pi/4}\right)^4-0.5357\left(\dfrac{\theta-\pi/2}{\pi/4}\right)^2+1, \quad \pi/2\le\theta\le3\pi/4 \\ -0.2991\left(\dfrac{\theta-3\pi/4}{\pi/4}\right)^5+0.2679\left(\dfrac{\theta-3\pi/4}{\pi/4}\right)^4+0.4911\left(\dfrac{\theta-3\pi/4}{\pi/4}\right)^3 \\ \qquad -0.0893\left(\dfrac{\theta-3\pi/4}{\pi/4}\right)^2-0.8705\left(\dfrac{\theta-3\pi/4}{\pi/4}\right)+0.5, \quad 3\pi/4\le\theta\le\pi \end{cases} \tag{f}$$

5 Figure 5-2 shows the displacement described by equation (f). The curves are very smooth, with very low values for velocity and acceleration. There is a jump discontinuity in jerk at the end knots, just as we knew there would be. This does not violate the fundamental law of cam design and is acceptable. The only complaint that might be suggested about this motion is that the jerk has a very high value at the jump, about 60 000 in/sec^3. Whether or not this is acceptable depends upon the application, of course, but assume it is not. This gives us a chance to show the real power of splines in cam design as well as what we mean by being able to control things with the extra knots. The jerk is high because we wanted to minimize the acceleration and the maximum velocity. We did this with those two interior knots at 45° and at 135°. The reasoning was that any displacement that starts at 0 at 0° and ends at 1 inch at 90° must have an average velocity of 1/90 = 0.01111 inches per degree. Any spline that we come up with will have this average. By placing a knot exactly half way between 0° and 90° with a displacement value equal to 0.5, we maintained that average both left and right of the knot. We did a similar thing at 135°.

6 We can slow things down for the initial and final parts of the motion by changing the values at those knots from 0.5 inches to something less, say 0.25 inches. Of course this means that the average velocity in the interval from 45° to 90° must increase as will the peak velocity from 90° to 135°. The accelerations will also increase. Table 5-3 lists these new conditions. Actually we only need to alter two equations from the set in equation (d).

$$\begin{aligned} a_1+b_1+c_1+d_1+e_1+f_1&=0.25 \\ a_2+b_2+c_2+d_2+e_2+f_2&=1 \\ a_3+b_3+c_3+d_3+e_3+f_3&=0.25 \end{aligned} \tag{g}$$

7 The results from the second trial spline can be seen in Figure 5-3. We have managed to lower the maximum jerk to about half of what it was and to make it continuous, just by tweaking a couple of knots. The acceleration curve now varies between –922 in/sec^2 and 500 in/sec^2, which may or may not be satisfactory. Again it depends on the application. It